

The LHC Higgs Boson Discovery: Implications for Finite Unified Theories

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Abstract

Finite Unified Theories (FUTs) are $N = 1$ supersymmetric Grand Unified Theories (GUTs) which can be made finite to all-loop orders, based on the principle of reduction of couplings, and therefore are provided with a large predictive power. We confront the predictions of an $SU(5)$ FUT with the top and bottom quark masses and other low-energy experimental constraints, resulting in a relatively heavy SUSY spectrum, naturally consistent with the non-observation of those particles at the LHC. The light Higgs boson mass is automatically predicted in the range compatible with the Higgs discovery at the LHC. Requiring a light Higgs-boson mass in the precise range of $M_h = 125.6 \pm 2.1$ GeV favors the lower part of the allowed spectrum, resulting in clear predictions for the discovery potential at current and future pp , as well as future e^+e^- colliders.

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1 The basic ideas

A large and sustained effort has been done in the recent years aiming to achieve a unified description of all interactions. Out of this endeavor two main directions have emerged as the most promising to attack the problem, namely, the superstring theories and non-commutative geometry. The two approaches, although at a different stage of development, have common unification targets and share similar hopes for exhibiting improved renormalization properties in the ultraviolet (UV) as compared to ordinary field theories. Moreover the two frameworks came closer by the observation that a natural realization of non-commutativity of space appears in the string theory context of D-branes in the presence of a constant background antisymmetric field [1]. Among the numerous important developments in both frameworks, it is worth noting two conjectures of utmost importance that signal the developments in certain directions in string theory, although not exclusively there, related to the main theme of the present review. The conjectures refer to (i) the duality among the 4-dimensional $N = 4$ supersymmetric Yang-Mills theory and the type IIB string theory on $AdS_5 \times S^5$ [2]; the former being the maximal $N = 4$ supersymmetric Yang-Mills theory is known to be UV all-loop finite theory [3, 4], (ii) the possibility of “miraculous” UV divergence cancellations in 4-dimensional maximal $N = 8$ supergravity leading to a finite theory, as has been confirmed in a remarkable 4-loop calculation [5–9]. However, despite the importance of having frameworks to discuss quantum gravity in a self-consistent way and possibly to construct finite theories in these type of frameworks, it is also very interesting to search for the minimal realistic framework in which finiteness can take place. After all the history of our field teaches us that if a new idea works, it does that in its simplest form. In addition, the main goal expected from a unified description of interactions by the particle physics community is to understand the present day large number of free parameters of the Standard Model (SM) in terms of a few fundamental ones. In other words, to achieve *reduction of couplings* at a more fundamental level.

To reduce the number of free parameters of a theory, and thus render it more predictive, one is usually led to introduce a symmetry. Grand Unified Theories (GUTs) are very good examples of such a procedure [10–14]. For instance, in the case of minimal $SU(5)$, because of (approximate) gauge coupling unification, it is possible to reduce the gauge couplings of the SM by one and give a prediction for one of them. In fact, LEP data [15] seem to suggest that a further symmetry, namely $N = 1$ global supersymmetry (SUSY) [16, 17] should also be required to make the prediction viable. GUTs can also relate the Yukawa couplings among themselves, again $SU(5)$ provided an example of this by predicting the ratio M_τ/M_b [18] in the SM. Unfortunately, requiring more gauge symmetry does not seem to help, since additional complications are introduced due to new degrees of freedom, in the ways and channels of breaking the symmetry, and so on.

A natural extension of the GUT idea is to find a way to relate the gauge and Yukawa sectors of a theory, that is to achieve Gauge-Yukawa Unification (GYU) [19–21]. A symmetry which naturally relates the two sectors is supersymmetry, in particular $N = 2$ SUSY [22]. It turns out, however, that $N = 2$ supersymmetric theories have serious phenomenological problems due to light mirror fermions. Also in superstring theories and in composite models there exist relations among the gauge and Yukawa couplings, but both kind of theories have phenomenological problems, which we are not going to address here.

In our studies [19–21, 23–28] we have developed a complementary strategy in searching for a more fundamental theory possibly at the Planck scale, whose basic ingredients are GUTs and supersymmetry, but its consequences certainly go beyond the known ones. Our method consists of hunting for renormalization group invariant (RGI) relations holding below the Planck scale, which in turn are preserved down to the GUT scale. This program, called Gauge–Yukawa unification scheme, applied in the dimensionless couplings of supersymmetric GUTs, such as gauge and Yukawa couplings, had already noticable successes by predicting correctly, among others, the top quark mass in the finite and in the minimal $N = 1$ supersymmetric SU(5) GUTs [23–25]. An impressive aspect of the RGI relations is that one can guarantee their validity to all-orders in perturbation theory by studying the uniqueness of the resulting relations at one-loop, as was proven [29, 30] in the early days of the program of *reduction of couplings* [29–34]. Even more remarkable is the fact that it is possible to find RGI relations among couplings that guarantee finiteness to all-orders in perturbation theory [35–39].

It is worth noting that the above principles have only been applied in supersymmetric GUTs for reasons that will be transparent in the following sections. We should also stress that our conjecture for GYU is by no means in conflict with earlier interesting proposals, but it rather uses all of them, hopefully in a more successful perspective. For instance, the use of SUSY GUTs comprises the demand of the cancellation of quadratic divergences in the SM. Similarly, the very interesting conjectures about the infrared fixed points are generalized in our proposal, since searching for RGI relations among various couplings corresponds to searching for *fixed points or surfaces* of the coupled differential equations obeyed by the various couplings of a theory.

Although SUSY seems to be an essential feature for a successful realization of the above program, its breaking has to be understood too, since it has the ambition to supply the non-SUSY SM with predictions for several of its free parameters. Indeed, the search for RGI relations has been extended to the soft SUSY breaking sector (SSB) of these theories [28, 40], which involves parameters of dimension one and two. A breakthrough concerning the renormalization properties of the SSB was made [41–47], based conceptually and technically on the work of Ref. [48]: the powerful supergraph method [49–52] for studying supersymmetric theories was applied to the softly broken ones by using the “spurion” external space-time independent superfields [53]. In the latter method a softly broken supersymmetric gauge theory is considered as a supersymmetric one in which the various parameters such as couplings and masses have been promoted to external superfields that acquire “vacuum expectation values”. Based on this method the relations among the soft term renormalization and that of an unbroken supersymmetric theory were derived. In particular the β -functions of the parameters of the softly broken theory are expressed in terms of partial differential operators involving the dimensionless parameters of the unbroken theory. The key point in the strategy of Refs. [44–47] in solving the set of coupled differential equations so as to be able to express all parameters in a RGI way, was to transform the partial differential operators involved to total derivative operators. This is indeed possible to be done on the RGI surface which is defined by the solution of the reduction equations.

On the phenomenological side there exist some serious developments, too. Previously an appealing “universal” set of soft scalar masses was assumed in the SSB sector of supersymmetric theories, given that apart from economy and simplicity (1) they are part of

the constraints that preserve finiteness up to two-loops [54, 55], (2) they are RGI up to two-loops in more general supersymmetric gauge theories, subject to the condition known as $P = 1/3 Q$ [40], and (3) they appear in the attractive dilaton dominated SUSY breaking superstring scenarios [56–58]. However, further studies have exhibited a number of problems all due to the restrictive nature of the “universality” assumption for the soft scalar masses. For instance, (a) in finite unified theories the universality predicts that the lightest supersymmetric particle is a charged particle, namely the superpartner of the τ lepton $\tilde{\tau}$, (b) the Minimal Supersymmetric Standard Model (MSSM) with universal soft scalar masses is inconsistent with the attractive radiative electroweak symmetry breaking [58], and (c) which is the worst of all, the universal soft scalar masses lead to charge and/or color breaking minima deeper than the standard vacuum [59]. Therefore, there have been attempts to relax this constraint without losing its attractive features. First, an interesting observation was made that in $N = 1$ Gauge–Yukawa unified theories there exists a RGI sum rule for the soft scalar masses at lower orders; at one-loop for the non-finite case [60] and at two-loops for the finite case [61]. The sum rule manages to overcome the above unpleasant phenomenological consequences. Moreover it was proven [47] that the sum rule for the soft scalar masses is RGI to all-orders for both the general as well as for the finite case. Finally, the exact β -function for the soft scalar masses in the Novikov-Shifman-Vainshtein-Zakharov (NSVZ) scheme [62–64] for the softly broken supersymmetric QCD has been obtained [47].

Armed with the above tools and results we are in a position to study and predict the spectrum of the full finite models in terms of few input parameters. In particular, a prediction for the lightest MSSM Higgs boson can be obtained. It turned out that the prediction is naturally in very good agreement with the discovery of a Higgs-like particle at the LHC [65, 66] at around ~ 126 GeV. Identifying the lightest Higgs boson with the newly discovered state one can restrict the allowed parameter space of the model. We review how this reduction of parameter space impacts the prediction of the SUSY spectrum and the discovery potential of the LHC and future e^+e^- colliders.

2 Theoretical basis

In this section we outline the idea of reduction of couplings. Any RGI relation among couplings (which does not depend on the renormalization scale μ explicitly) can be expressed, in the implicit form $\Phi(g_1, \dots, g_A) = \text{const.}$, which has to satisfy the partial differential equation (PDE)

$$\mu \frac{d\Phi}{d\mu} = \vec{\nabla} \cdot \vec{\beta} = \sum_{a=1}^A \beta_a \frac{\partial \Phi}{\partial g_a} = 0, \quad (1)$$

where β_a is the β -function of g_a . This PDE is equivalent to a set of ordinary differential equations, the so-called reduction equations (REs) [29, 30, 67],

$$\beta_g \frac{dg_a}{dg} = \beta_a, \quad a = 1, \dots, A, \quad (2)$$

where g and β_g are the primary coupling and its β -function, and the counting on a does not include g . Since maximally $(A - 1)$ independent RGI “constraints” in the A -dimensional

space of couplings can be imposed by the Φ_a 's, one could in principle express all the couplings in terms of a single coupling g . However, a closer look to the set of Eqs. (2) reveals that their general solutions contain as many integration constants as the number of equations themselves. Thus, using such integration constants we have just traded an integration constant for each ordinary renormalized coupling, and consequently, these general solutions cannot be considered as reduced ones. The crucial requirement in the search for RGE relations is to demand power series solutions to the REs,

$$g_a = \sum_n \rho_a^{(n)} g^{2n+1}, \quad (3)$$

which preserve perturbative renormalizability. Such an ansatz fixes the corresponding integration constant in each of the REs and picks up a special solution out of the general one. Remarkably, the uniqueness of such power series solutions can be decided already at the one-loop level [29,30,67]. To illustrate this, let us assume that the β -functions have the form

$$\begin{aligned} \beta_a &= \frac{1}{16\pi^2} \left[\sum_{b,c,d \neq g} \beta_a^{(1)bcd} g_b g_c g_d + \sum_{b \neq g} \beta_a^{(1)b} g_b g^2 \right] + \dots, \\ \beta_g &= \frac{1}{16\pi^2} \beta_g^{(1)} g^3 + \dots, \end{aligned} \quad (4)$$

where \dots stands for higher order terms, and $\beta_a^{(1)bcd}$'s are symmetric in b, c, d . We then assume that the $\rho_a^{(n)}$'s with $n \leq r$ have been uniquely determined. To obtain $\rho_a^{(r+1)}$'s, we insert the power series (3) into the REs (2) and collect terms of $\mathcal{O}(g^{2r+3})$ and find

$$\sum_{d \neq g} M(r)_a^d \rho_d^{(r+1)} = \text{lower order quantities},$$

where the r.h.s. is known by assumption, and

$$M(r)_a^d = 3 \sum_{b,c \neq g} \beta_a^{(1)bcd} \rho_b^{(1)} \rho_c^{(1)} + \beta_a^{(1)d} - (2r+1) \beta_g^{(1)} \delta_a^d, \quad (5)$$

$$0 = \sum_{b,c,d \neq g} \beta_a^{(1)bcd} \rho_b^{(1)} \rho_c^{(1)} \rho_d^{(1)} + \sum_{d \neq g} \beta_a^{(1)d} \rho_d^{(1)} - \beta_g^{(1)} \rho_a^{(1)}, \quad (6)$$

Therefore, the $\rho_a^{(n)}$'s for all $n > 1$ for a given set of $\rho_a^{(1)}$'s can be uniquely determined if $\det M(n)_a^d \neq 0$ for all $n \geq 0$.

As it will be clear later by examining specific examples, the various couplings in supersymmetric theories have the same asymptotic behaviour. Therefore searching for a power series solution of the form (3) to the REs (2) is justified. This is not the case in non-supersymmetric theories, although the deeper reason for this fact is not fully understood.

The possibility of coupling unification described in this section is without any doubt attractive because the ‘‘completely reduced’’ theory contains only one independent coupling, but it can be unrealistic. Therefore, one often would like to impose fewer RGI constraints, and this is the idea of partial reduction [68,69].

2.1 Reduction of dimensionful parameters

The reduction of couplings was originally formulated for massless theories on the basis of the Callan-Symanzik equation [29, 30, 67]. The extension to theories with massive parameters is not straightforward if one wants to keep the generality and the rigor on the same level as for the massless case; one has to fulfill a set of requirements coming from the renormalization group equations, the Callan-Symanzik equations, etc. along with the normalization conditions imposed on irreducible Green's functions [70]. See [71] for interesting results in this direction. Here to simplify the situation and following Refs. [28, 29] we would like to assume that a mass-independent renormalization scheme has been employed so that all the RG functions have only trivial dependencies of dimensional parameters.

To be general, we consider a renormalizable theory which contains a set of $(N + 1)$ dimension-zero couplings, $\{\hat{g}_0, \hat{g}_1, \dots, \hat{g}_N\}$, as well as a set of L parameters with dimension one, $\{\hat{h}_1, \dots, \hat{h}_L\}$, and a set of M parameters with dimension two, $\{\hat{m}_1^2, \dots, \hat{m}_M^2\}$. The renormalized irreducible vertex function satisfies the RG equation

$$\begin{aligned} 0 &= \mathcal{D}\Gamma[\Phi's; \hat{g}_0, \hat{g}_1, \dots, \hat{g}_N; \hat{h}_1, \dots, \hat{h}_L; \hat{m}_1^2, \dots, \hat{m}_M^2; \mu] , \\ \mathcal{D} &= \mu \frac{\partial}{\partial \mu} + \sum_{i=0}^N \beta_i \frac{\partial}{\partial \hat{g}_i} + \sum_{a=1}^L \gamma_a^h \frac{\partial}{\partial \hat{h}_a} + \sum_{\alpha=1}^M \gamma_\alpha^{m^2} \frac{\partial}{\partial \hat{m}_\alpha^2} + \sum_J \Phi_I \gamma^{\phi I}_J \frac{\delta}{\delta \Phi_J} . \end{aligned} \quad (7)$$

Since we assume a mass-independent renormalization scheme, the γ 's have the form

$$\begin{aligned} \gamma_a^h &= \sum_{b=1}^L \gamma_a^{h,b}(g_0, \dots, g_N) \hat{h}_b , \\ \gamma_\alpha^{m^2} &= \sum_{\beta=1}^M \gamma_\alpha^{m^2,\beta}(g_0, \dots, g_N) \hat{m}_\beta^2 + \sum_{a,b=1}^L \gamma_\alpha^{m^2,ab}(g_0, \dots, g_N) \hat{h}_a \hat{h}_b , \end{aligned} \quad (8)$$

where $\gamma_a^{h,b}, \gamma_\alpha^{m^2,\beta}$ and $\gamma_\alpha^{m^2,ab}$ are power series of the dimension-zero couplings g 's in perturbation theory.

As in the massless case, we then look for conditions under which the reduction of parameters,

$$\hat{g}_i = \hat{g}_i(g) , \quad (i = 1, \dots, N) , \quad (9)$$

$$\hat{h}_a = \sum_{b=1}^P f_a^b(g) h_b , \quad (a = P + 1, \dots, L) , \quad (10)$$

$$\hat{m}_\alpha^2 = \sum_{\beta=1}^Q e_\alpha^\beta(g) m_\beta^2 + \sum_{a,b=1}^P k_\alpha^{ab}(g) h_a h_b , \quad (\alpha = Q + 1, \dots, M) , \quad (11)$$

is consistent with the RG equation (1), where we assume that $g \equiv g_0$, $h_a \equiv \hat{h}_a$ ($1 \leq a \leq P$) and $m_\alpha^2 \equiv \hat{m}_\alpha^2$ ($1 \leq \alpha \leq Q$) are independent parameters of the reduced theory. We find that the following set of equations has to be satisfied:

$$\beta_g \frac{\partial \hat{g}_i}{\partial g} = \beta_i , \quad (i = 1, \dots, N) , \quad (12)$$

$$\beta_g \frac{\partial \hat{h}_a}{\partial g} + \sum_{b=1}^P \gamma_b^h \frac{\partial \hat{h}_a}{\partial h_b} = \gamma_a^h, \quad (a = P+1, \dots, L), \quad (13)$$

$$\beta_g \frac{\partial \hat{m}_\alpha^2}{\partial g} + \sum_{a=1}^P \gamma_a^h \frac{\partial \hat{m}_\alpha^2}{\partial h_a} + \sum_{\beta=1}^Q \gamma_\beta^{m^2} \frac{\partial \hat{m}_\alpha^2}{\partial m_\beta^2} = \gamma_\alpha^{m^2}, \quad (\alpha = Q+1, \dots, M). \quad (14)$$

Using Eq. (8) for γ 's, one finds that eqs.(12-14) reduce to

$$\beta_g \frac{df_a^b}{dg} + \sum_{c=1}^P f_a^c [\gamma_c^{h,b} + \sum_{d=P+1}^L \gamma_c^{h,d} f_d^b] - \gamma_a^{h,b} - \sum_{d=P+1}^L \gamma_a^{h,d} f_d^b = 0, \quad (15)$$

$$(a = P+1, \dots, L; b = 1, \dots, P),$$

$$\beta_g \frac{de_\alpha^\beta}{dg} + \sum_{\gamma=1}^Q e_\alpha^\gamma [\gamma_\gamma^{m^2,\beta} + \sum_{\delta=Q+1}^M \gamma_\gamma^{m^2,\delta} e_\delta^\beta] - \gamma_\alpha^{m^2,\beta} - \sum_{\delta=Q+1}^M \gamma_\alpha^{m^2,\delta} e_\delta^\beta = 0, \quad (16)$$

$$(\alpha = Q+1, \dots, M; \beta = 1, \dots, Q),$$

$$\begin{aligned} \beta_g \frac{dk_\alpha^{ab}}{dg} + 2 \sum_{c=1}^P (\gamma_c^{h,a} + \sum_{d=P+1}^L \gamma_c^{h,d} f_d^a) k_\alpha^{cb} + \sum_{\beta=1}^Q e_\alpha^\beta [\gamma_\beta^{m^2,ab} + \sum_{c,d=P+1}^L \gamma_\beta^{m^2,cd} f_c^a f_d^b \\ + 2 \sum_{c=P+1}^L \gamma_\beta^{m^2,cb} f_c^a + \sum_{\delta=Q+1}^M \gamma_\beta^{m^2,\delta} k_\delta^{ab}] - [\gamma_\alpha^{m^2,ab} + \sum_{c,d=P+1}^L \gamma_\alpha^{m^2,cd} f_c^a f_d^b \\ + 2 \sum_{c=P+1}^L \gamma_\alpha^{m^2,cb} f_c^a + \sum_{\delta=Q+1}^M \gamma_\alpha^{m^2,\delta} k_\delta^{ab}] = 0, \quad (17) \\ (\alpha = Q+1, \dots, M; a, b = 1, \dots, P). \end{aligned}$$

If these equations are satisfied, the irreducible vertex function of the reduced theory

$$\begin{aligned} \Gamma_R[\Phi' s; g; h_1, \dots, h_P; m_1^2, \dots, \hat{m}_Q^2; \mu] \\ \equiv \Gamma[\Phi' s; g; \hat{g}_1(g), \dots, \hat{g}_N(g); h_1, \dots, h_P, \hat{h}_{P+1}(g, h), \dots, \hat{h}_L(g, h); \\ m_1^2, \dots, \hat{m}_Q^2, \hat{m}_{Q+1}^2(g, h, m^2), \dots, \hat{m}_M^2(g, h, m^2); \mu] \end{aligned} \quad (18)$$

has the same renormalization group flow as the original one.

The requirement for the reduced theory to be perturbative renormalizable means that the functions \hat{g}_i , f_a^b , e_α^β and k_α^{ab} , defined in eqs. (9-11), should have a power series expansion in the primary coupling g :

$$\begin{aligned} \hat{g}_i &= g \sum_{n=0}^{\infty} \rho_i^{(n)} g^n, \quad f_a^b = g \sum_{n=0}^{\infty} \eta_a^{b(n)} g^n, \\ e_\alpha^\beta &= \sum_{n=0}^{\infty} \xi_\alpha^{\beta(n)} g^n, \quad k_\alpha^{ab} = \sum_{n=0}^{\infty} \chi_\alpha^{ab(n)} g^n. \end{aligned} \quad (19)$$

To obtain the expansion coefficients, we insert the power series ansatz above into eqs. (12,15-17) and require that the equations are satisfied at each order in g . Note that the existence of a unique power series solution is a non-trivial matter: it depends on the theory as well as on the choice of the set of independent parameters.

2.2 Finiteness in N=1 Supersymmetric Gauge Theories

Let us consider a chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g . The superpotential of the theory is given by

$$W = \frac{1}{2} m_{ij} \phi_i \phi_j + \frac{1}{6} C_{ijk} \phi_i \phi_j \phi_k , \quad (20)$$

where m_{ij} and C_{ijk} are gauge invariant tensors and the matter field ϕ_i transforms according to the irreducible representation R_i of the gauge group G . The renormalization constants associated with the superpotential (20), assuming that SUSY is preserved, are

$$\phi_i^0 = (Z_i^j)^{(1/2)} \phi_j , \quad (21)$$

$$m_{ij}^0 = Z_{ij}^{i'j'} m_{i'j'} , \quad (22)$$

$$C_{ijk}^0 = Z_{ijk}^{i'j'k'} C_{i'j'k'} . \quad (23)$$

The $N = 1$ non-renormalization theorem [51, 72, 73] ensures that there are no mass and cubic-interaction-term infinities and therefore

$$\begin{aligned} Z_{ijk}^{i'j'k'} Z_{i'}^{1/2 i''} Z_{j'}^{1/2 j''} Z_{k'}^{1/2 k''} &= \delta_{(i}^{i''} \delta_j^{j''} \delta_{k)}^{k''} , \\ Z_{ij}^{i'j'} Z_{i'}^{1/2 i''} Z_{j'}^{1/2 j''} &= \delta_{(i}^{i''} \delta_{j)}^{j''} . \end{aligned} \quad (24)$$

As a result the only surviving possible infinities are the wave-function renormalization constants Z_i^j , i.e., one infinity for each field. The one-loop β -function of the gauge coupling g is given by [74]

$$\beta_g^{(1)} = \frac{dg}{dt} = \frac{g^3}{16\pi^2} \left[\sum_i l(R_i) - 3 C_2(G) \right] , \quad (25)$$

where $l(R_i)$ is the Dynkin index of R_i and $C_2(G)$ is the quadratic Casimir invariant of the adjoint representation of the gauge group G . The β -functions of C_{ijk} , by virtue of the non-renormalization theorem, are related to the anomalous dimension matrix γ_{ij} of the matter fields ϕ_i as:

$$\beta_{ijk} = \frac{dC_{ijk}}{dt} = C_{ijl} \gamma_k^l + C_{ikl} \gamma_j^l + C_{jkl} \gamma_i^l . \quad (26)$$

At one-loop level γ_{ij} is [74]

$$\gamma_j^{i(1)} = \frac{1}{32\pi^2} [C^{ikl} C_{jkl} - 2 g^2 C_2(R) \delta_j^i] , \quad (27)$$

where $C_2(R)$ is the quadratic Casimir invariant of the representation R_i , and $C^{ijk} = C_{ijk}^*$. Since dimensional coupling parameters such as masses and couplings of cubic scalar field terms do not influence the asymptotic properties of a theory on which we are interested here, it is sufficient to take into account only the dimensionless supersymmetric couplings such as g and C_{ijk} . So we neglect the existence of dimensional parameters, and assume furthermore that C_{ijk} are real so that C_{ijk}^2 are always positive numbers.

As one can see from Eqs. (25) and (27), all the one-loop β -functions of the theory vanish if $\beta_g^{(1)}$ and $\gamma_{ij}^{(1)}$ vanish, i.e.

$$\sum_i \ell(R_i) = 3C_2(G), \quad (28)$$

$$C^{ikl}C_{jkl} = 2\delta_j^i g^2 C_2(R_i), \quad (29)$$

The conditions for finiteness for $N = 1$ field theories with $SU(N)$ gauge symmetry are discussed in [75], and the analysis of the anomaly-free and no-charge renormalization requirements for these theories can be found in [76]. A very interesting result is that the conditions (28,29) are necessary and sufficient for finiteness at the two-loop level [74,77–80].

In case SUSY is broken by soft terms, the requirement of finiteness in the one-loop soft breaking terms imposes further constraints among themselves [54]. In addition, the same set of conditions that are sufficient for one-loop finiteness of the soft breaking terms render the soft sector of the theory two-loop finite [55].

The one- and two-loop finiteness conditions (28,29) restrict considerably the possible choices of the irreducible representations (irreps) R_i for a given group G as well as the Yukawa couplings in the superpotential (20). Note in particular that the finiteness conditions cannot be applied to the minimal supersymmetric standard model (MSSM), since the presence of a $U(1)$ gauge group is incompatible with the condition (28), due to $C_2[U(1)] = 0$. This naturally leads to the expectation that finiteness should be attained at the grand unified level only, the MSSM being just the corresponding, low-energy, effective theory.

Another important consequence of one- and two-loop finiteness is that SUSY (most probably) can only be broken due to the soft breaking terms. Indeed, due to the unacceptability of gauge singlets, F-type spontaneous symmetry breaking [81] terms are incompatible with finiteness, as well as D-type [82] spontaneous breaking which requires the existence of a $U(1)$ gauge group.

A natural question to ask is what happens at higher loop orders. The answer is contained in a theorem [35,36] which states the necessary and sufficient conditions to achieve finiteness at all orders. Before we discuss the theorem let us make some introductory remarks. The finiteness conditions impose relations between gauge and Yukawa couplings. To require such relations which render the couplings mutually dependent at a given renormalization point is trivial. What is not trivial is to guarantee that relations leading to a reduction of the couplings hold at any renormalization point. As we have seen, the necessary and also sufficient, condition for this to happen is to require that such relations are solutions to the REs

$$\beta_g \frac{dC_{ijk}}{dg} = \beta_{ijk} \quad (30)$$

and hold at all orders. Remarkably, the existence of all-order power series solutions to (30) can be decided at one-loop level, as already mentioned.

Let us now turn to the all-order finiteness theorem [35,36], which states that if an $N = 1$ supersymmetric gauge theory can become finite to all orders in the sense of vanishing β -functions, that is of physical scale invariance. It is based on (a) the structure of the supercurrent in $N = 1$ supersymmetric gauge theory [83–85], and on (b) the non-renormalization properties of $N = 1$ chiral anomalies [35,36,86–88]. Details on the proof can be found in

refs. [35, 36] and further discussion in Refs. [37, 86–89]. Here, following mostly Ref. [89] we present a comprehensible sketch of the proof.

Consider an $N = 1$ supersymmetric gauge theory, with simple Lie group G . The content of this theory is given at the classical level by the matter supermultiplets S_i , which contain a scalar field ϕ_i and a Weyl spinor ψ_{ia} , and the vector supermultiplet V_a , which contains a gauge vector field A_μ^a and a gaugino Weyl spinor λ_α^a .

Let us first recall certain facts about the theory:

(1) A massless $N = 1$ supersymmetric theory is invariant under a $U(1)$ chiral transformation R under which the various fields transform as follows

$$\begin{aligned} A'_\mu &= A_\mu, \quad \lambda'_\alpha = \exp(-i\theta)\lambda_\alpha \\ \phi' &= \exp(-i\frac{2}{3}\theta)\phi, \quad \psi'_\alpha = \exp(-i\frac{1}{3}\theta)\psi_\alpha, \quad \dots \end{aligned} \quad (31)$$

The corresponding axial Noether current $J_R^\mu(x)$ is

$$J_R^\mu(x) = \bar{\lambda}\gamma^\mu\gamma^5\lambda + \dots \quad (32)$$

is conserved classically, while in the quantum case is violated by the axial anomaly

$$\partial_\mu J_R^\mu = r(\epsilon^{\mu\nu\sigma\rho}F_{\mu\nu}F_{\sigma\rho} + \dots). \quad (33)$$

From its known topological origin in ordinary gauge theories [90–92], one would expect the axial vector current J_R^μ to satisfy the Adler-Bardeen theorem and receive corrections only at the one-loop level. Indeed it has been shown that the same non-renormalization theorem holds also in supersymmetric theories [86–88]. Therefore

$$r = \hbar\beta_g^{(1)}. \quad (34)$$

(2) The massless theory we consider is scale invariant at the classical level and, in general, there is a scale anomaly due to radiative corrections. The scale anomaly appears in the trace of the energy momentum tensor $T_{\mu\nu}$, which is traceless classically. It has the form

$$T_\mu^\mu = \beta_g F^{\mu\nu}F_{\mu\nu} + \dots \quad (35)$$

(3) Massless, $N = 1$ supersymmetric gauge theories are classically invariant under the supersymmetric extension of the conformal group – the superconformal group. Examining the superconformal algebra, it can be seen that the subset of superconformal transformations consisting of translations, SUSY transformations, and axial R transformations is closed under SUSY, i.e. these transformations form a representation of SUSY. It follows that the conserved currents corresponding to these transformations make up a supermultiplet represented by an axial vector superfield called the supercurrent J ,

$$J \equiv \{J_R^\mu, Q_\alpha^\mu, T_\nu^\mu, \dots\}, \quad (36)$$

where J_R^μ is the current associated to R invariance, Q_α^μ is the one associated to SUSY invariance, and T_ν^μ the one associated to translational invariance (energy-momentum tensor).

The anomalies of the R current J_R^μ , the trace anomalies of the SUSY current, and the energy-momentum tensor, form also a second supermultiplet, called the supertrace anomaly

$$S = \{Re S, Im S, S_\alpha\} = \{T_\mu^\mu, \partial_\mu J_R^\mu, \sigma_{\alpha\dot{\beta}}^\mu \bar{Q}_\mu^{\dot{\beta}} + \dots\} \quad (37)$$

where T_μ^μ in Eq.(35) and

$$\partial_\mu J_R^\mu = \beta_g \epsilon^{\mu\nu\sigma\rho} F_{\mu\nu} F_{\sigma\rho} + \dots \quad (38)$$

$$\sigma_{\alpha\dot{\beta}}^\mu \bar{Q}_\mu^{\dot{\beta}} = \beta_g \lambda^\beta \sigma_{\alpha\dot{\beta}}^{\mu\nu} F_{\mu\nu} + \dots \quad (39)$$

(4) It is very important to note that the Noether current defined in (32) is not the same as the current associated to R invariance that appears in the supercurrent J in (36), but they coincide in the tree approximation. So starting from a unique classical Noether current $J_{R(class)}^\mu$, the Noether current J_R^μ is defined as the quantum extension of $J_{R(class)}^\mu$ which allows for the validity of the non-renormalization theorem. On the other hand $J_R'^\mu$, is defined to belong to the supercurrent J , together with the energy-momentum tensor. The two requirements cannot be fulfilled by a single current operator at the same time.

Although the Noether current J_R^μ which obeys (33) and the current $J_R'^\mu$ belonging to the supercurrent multiplet J are not the same, there is a relation [35, 36] between quantities associated with them

$$r = \beta_g(1 + x_g) + \beta_{ijk} x^{ijk} - \gamma_A r^A \quad (40)$$

where r was given in Eq. (34). The r^A are the non-renormalized coefficients of the anomalies of the Noether currents associated to the chiral invariances of the superpotential, and – like r – are strictly one-loop quantities. The γ_A 's are linear combinations of the anomalous dimensions of the matter fields, and x_g , and x^{ijk} are radiative correction quantities. The structure of equality (40) is independent of the renormalization scheme.

One-loop finiteness, i.e. vanishing of the β -functions at one-loop, implies that the Yukawa couplings λ_{ijk} must be functions of the gauge coupling g . To find a similar condition to all orders it is necessary and sufficient for the Yukawa couplings to be a formal power series in g , which is solution of the REs (30).

We can now state the theorem for all-order vanishing β -functions.

Theorem:

Consider an $N = 1$ supersymmetric Yang-Mills theory, with simple gauge group. If the following conditions are satisfied

1. There is no gauge anomaly.
2. The gauge β -function vanishes at one-loop

$$\beta_g^{(1)} = 0 = \sum_i l(R_i) - 3 C_2(G). \quad (41)$$

3. There exist solutions of the form

$$C_{ijk} = \rho_{ijk} g, \quad \rho_{ijk} \in \mathbb{C} \quad (42)$$

to the conditions of vanishing one-loop matter fields anomalous dimensions

$$\begin{aligned} \gamma_j^{i(1)} &= 0 \\ &= \frac{1}{32\pi^2} [C^{ikl} C_{jkl} - 2 g^2 C_2(R) \delta_j^i]. \end{aligned} \quad (43)$$

4. These solutions are isolated and non-degenerate when considered as solutions of vanishing one-loop Yukawa β -functions:

$$\beta_{ijk} = 0. \quad (44)$$

Then, each of the solutions (42) can be uniquely extended to a formal power series in g , and the associated super Yang-Mills models depend on the single coupling constant g with a β function which vanishes at all-orders.

It is important to note a few things: The requirement of isolated and non-degenerate solutions guarantees the existence of a unique formal power series solution to the reduction equations. The vanishing of the gauge β function at one-loop, $\beta_g^{(1)}$, is equivalent to the vanishing of the R current anomaly (33). The vanishing of the anomalous dimensions at one-loop implies the vanishing of the Yukawa couplings β functions at that order. It also implies the vanishing of the chiral anomaly coefficients r^A . This last property is a necessary condition for having β functions vanishing at all orders.¹

Proof:

Insert β_{ijk} as given by the REs into the relationship (40) between the axial anomalies coefficients and the β -functions. Since these chiral anomalies vanish, we get for β_g an homogeneous equation of the form

$$0 = \beta_g(1 + O(\hbar)). \quad (45)$$

The solution of this equation in the sense of a formal power series in \hbar is $\beta_g = 0$, order by order. Therefore, due to the REs (30), $\beta_{ijk} = 0$ too.

Thus we see that finiteness and reduction of couplings are intimately related. Since an equation like eq. (40) is lacking in non-supersymmetric theories, one cannot extend the validity of a similar theorem in such theories.

2.3 Sum rule for SB terms in $N = 1$ Supersymmetric and Finite theories: All-loop results

As we have seen in Sect. 2.1, the method of reducing the dimensionless couplings has been extended [28], to the soft SUSY breaking (SSB) dimensionful parameters of $N = 1$ supersymmetric theories. In addition it was found [60] that RGI SSB scalar masses in Gauge-Yukawa

¹There is an alternative way to find finite theories [93].

unified models satisfy a universal sum rule. Here we will describe first how the use of the available two-loop RG functions and the requirement of finiteness of the SSB parameters up to this order leads to the soft scalar-mass sum rule [61].

Consider the superpotential given by (20) along with the Lagrangian for SSB terms

$$\begin{aligned} -\mathcal{L}_{\text{SB}} &= \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j \\ &+ \frac{1}{2} (m^2)_i^j \phi^{*i} \phi_j + \frac{1}{2} M \lambda \lambda + \text{h.c.}, \end{aligned} \quad (46)$$

where the ϕ_i are the scalar parts of the chiral superfields Φ_i , λ are the gauginos and M their unified mass. Since we would like to consider only finite theories here, we assume that the gauge group is a simple group and the one-loop β -function of the gauge coupling g vanishes. We also assume that the reduction equations admit power series solutions of the form

$$C^{ijk} = g \sum_n \rho_{(n)}^{ijk} g^{2n}. \quad (47)$$

According to the finiteness theorem of Refs. [35, 36], the theory is then finite to all orders in perturbation theory, if, among others, the one-loop anomalous dimensions $\gamma_i^{j(1)}$ vanish. The one- and two-loop finiteness for h^{ijk} can be achieved by [55]

$$h^{ijk} = -MC^{ijk} + \dots = -M\rho_{(0)}^{ijk} g + O(g^5), \quad (48)$$

where \dots stand for higher order terms.

Now, to obtain the two-loop sum rule for soft scalar masses, we assume that the lowest order coefficients $\rho_{(0)}^{ijk}$ and also $(m^2)_j^i$ satisfy the diagonality relations

$$\rho_{ipq(0)} \rho_{(0)}^{jpq} \propto \delta_i^j \text{ for all } p \text{ and } q \text{ and } (m^2)_j^i = m_j^2 \delta_j^i, \quad (49)$$

respectively. Then we find the following soft scalar-mass sum rule [21, 61, 94]

$$(m_i^2 + m_j^2 + m_k^2)/MM^\dagger = 1 + \frac{g^2}{16\pi^2} \Delta^{(2)} + O(g^4) \quad (50)$$

for i, j, k with $\rho_{(0)}^{ijk} \neq 0$, where $\Delta^{(2)}$ is the two-loop correction

$$\Delta^{(2)} = -2 \sum_l [(m_l^2/MM^\dagger) - (1/3)] T(R_l), \quad (51)$$

which vanishes for the universal choice in accordance with the previous findings of Ref. [55] (in here $T(R_l)$ is the Dynkin index of the R_l irrep).

If we know higher-loop β -functions explicitly, we can follow the same procedure and find higher-loop RGI relations among SSB terms. However, the β -functions of the soft scalar masses are explicitly known only up to two loops. In order to obtain higher-loop results some relations among β -functions are needed.

Making use of the spurion technique [49–53], it is possible to find the following all-loop relations among SSB β -functions, [41–46]

$$\beta_M = 2\mathcal{O}\left(\frac{\beta_g}{g}\right), \quad (52)$$

$$\begin{aligned}\beta_h^{ijk} &= \gamma^i_l h^{ljk} + \gamma^j_l h^{ilk} + \gamma^k_l h^{ijl} \\ &\quad - 2\gamma^i_{1l} C^{ljk} - 2\gamma^j_{1l} C^{ilk} - 2\gamma^k_{1l} C^{ijl} ,\end{aligned}\tag{53}$$

$$(\beta_{m^2})^i_j = \left[\Delta + X \frac{\partial}{\partial g} \right] \gamma^i_j ,\tag{54}$$

$$\mathcal{O} = \left(M g^2 \frac{\partial}{\partial g^2} - h^{lmn} \frac{\partial}{\partial C^{lmn}} \right) ,\tag{55}$$

$$\Delta = 2\mathcal{O}\mathcal{O}^* + 2|M|^2 g^2 \frac{\partial}{\partial g^2} + \tilde{C}_{lmn} \frac{\partial}{\partial C_{lmn}} + \tilde{C}^{lmn} \frac{\partial}{\partial C^{lmn}} ,\tag{56}$$

where $(\gamma_1)^i_j = \mathcal{O}\gamma^i_j$, $C_{lmn} = (C^{lmn})^*$, and

$$\tilde{C}^{ijk} = (m^2)^i_l C^{ljk} + (m^2)^j_l C^{ilk} + (m^2)^k_l C^{ijl} .\tag{57}$$

It was also found [42] that the relation

$$h^{ijk} = -M(C^{ijk})' \equiv -M \frac{dC^{ijk}(g)}{d \ln g} ,\tag{58}$$

among couplings is all-loop RGI. Furthermore, using the all-loop gauge β -function of Novikov *et al.* [62–64] given by

$$\beta_g^{\text{NSVZ}} = \frac{g^3}{16\pi^2} \left[\frac{\sum_l T(R_l)(1 - \gamma_l/2) - 3C(G)}{1 - g^2 C(G)/8\pi^2} \right] ,\tag{59}$$

it was found the all-loop RGI sum rule [47],

$$\begin{aligned}m_i^2 + m_j^2 + m_k^2 &= |M|^2 \left\{ \frac{1}{1 - g^2 C(G)/(8\pi^2)} \frac{d \ln C^{ijk}}{d \ln g} + \frac{1}{2} \frac{d^2 \ln C^{ijk}}{d(\ln g)^2} \right\} \\ &\quad + \sum_l \frac{m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2} \frac{d \ln C^{ijk}}{d \ln g} .\end{aligned}\tag{60}$$

In addition the exact- β -function for m^2 in the NSVZ scheme has been obtained [47] for the first time and is given by

$$\begin{aligned}\beta_{m_i^2}^{\text{NSVZ}} &= \left[|M|^2 \left\{ \frac{1}{1 - g^2 C(G)/(8\pi^2)} \frac{d}{d \ln g} + \frac{1}{2} \frac{d^2}{d(\ln g)^2} \right\} \right. \\ &\quad \left. + \sum_l \frac{m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2} \frac{d}{d \ln g} \right] \gamma_i^{\text{NSVZ}} .\end{aligned}\tag{61}$$

Surprisingly enough, the all-loop result (60) coincides with the superstring result for the finite case in a certain class of orbifold models [61] if $d \ln C^{ijk}/d \ln g = 1$.

3 The (so far) best Finite Unified Theory

3.1 Definition of $SU(5)$ -FUT

We review an all-loop Finite Unified theory with $SU(5)$ as gauge group, where the reduction of couplings has been applied to the third generation of quarks and leptons. The particle

content of the model we will study, which we denote **$SU(5)$ -FUT** consists of the following supermultiplets: three ($\mathbf{\bar{5}} + \mathbf{10}$), needed for each of the three generations of quarks and leptons, four ($\mathbf{\bar{5}} + \mathbf{5}$) and one $\mathbf{24}$ considered as Higgs supermultiplets. When the gauge group of the finite GUT is broken the theory is no longer finite, and we will assume that we are left with the MSSM [20, 23, 25, 26, 95].

A predictive Gauge-Yukawa unified $SU(5)$ model which is finite to all orders, in addition to the requirements mentioned already, should also have the following properties:

1. One-loop anomalous dimensions are diagonal, i.e., $\gamma_i^{(1)j} \propto \delta_i^j$.
2. Three fermion generations, in the irreducible representations $\mathbf{\bar{5}}_i, \mathbf{10}_i$ ($i = 1, 2, 3$), which obviously should not couple to the adjoint $\mathbf{24}$.
3. The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs quintet and anti-quintet, which couple to the third generation.

After the reduction of couplings the symmetry is enhanced, leading to the following superpotential [61, 96]

$$W = \sum_{i=1}^3 \left[\frac{1}{2} g_i^u \mathbf{10}_i \mathbf{10}_i H_i + g_i^d \mathbf{10}_i \mathbf{\bar{5}}_i \overline{H}_i \right] + g_{23}^u \mathbf{10}_2 \mathbf{10}_3 H_4 \quad (62)$$

$$+ g_{23}^d \mathbf{10}_2 \mathbf{\bar{5}}_3 \overline{H}_4 + g_{32}^d \mathbf{10}_3 \mathbf{\bar{5}}_2 \overline{H}_4 + g_2^f H_2 \mathbf{24} \overline{H}_2 + g_3^f H_3 \mathbf{24} \overline{H}_3 + \frac{g^\lambda}{3} (\mathbf{24})^3 .$$

The non-degenerate and isolated solutions to $\gamma_i^{(1)} = 0$ give us:

$$(g_1^u)^2 = \frac{8}{5} g^2, \quad (g_1^d)^2 = \frac{6}{5} g^2, \quad (g_2^u)^2 = (g_3^u)^2 = \frac{4}{5} g^2, \quad (63)$$

$$(g_2^d)^2 = (g_3^d)^2 = \frac{3}{5} g^2, \quad (g_{23}^u)^2 = \frac{4}{5} g^2, \quad (g_{23}^d)^2 = (g_{32}^d)^2 = \frac{3}{5} g^2,$$

$$(g^\lambda)^2 = \frac{15}{7} g^2, \quad (g_2^f)^2 = (g_3^f)^2 = \frac{1}{2} g^2, \quad (g_1^f)^2 = 0, \quad (g_4^f)^2 = 0,$$

and from the sum rule we obtain:

$$m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2, \quad m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3}, \quad m_{\mathbf{\bar{5}}}^2 + 3m_{\mathbf{10}}^2 = \frac{4M^2}{3}, \quad (64)$$

i.e., in this case we have only two free parameters $m_{\mathbf{10}}$ and M for the dimensionful sector.

As already mentioned, after the $SU(5)$ gauge symmetry breaking we assume we have the MSSM, i.e. only two Higgs doublets. This can be achieved by introducing appropriate mass terms that allow to perform a rotation of the Higgs sector [23, 24, 97–99], in such a way that only one pair of Higgs doublets, coupled mostly to the third family, remains light and acquire vacuum expectation values. To avoid fast proton decay the usual fine tuning to achieve doublet-triplet splitting is performed, although the mechanism is not identical to minimal $SU(5)$, since we have an extended Higgs sector.

Thus, after the gauge symmetry of the GUT theory is broken we are left with the MSSM, with the boundary conditions for the third family given by the finiteness conditions, while the other two families are not restricted.

3.2 Predictions of the Finite Model for quark masses and other experimental constraints

Since the gauge symmetry is spontaneously broken below M_{GUT} , the finiteness conditions do not restrict the renormalization properties at low energies, and all it remains are boundary conditions on the gauge and Yukawa couplings (63), the $h = -MC$ (48) relation, and the soft scalar-mass sum rule at M_{GUT} .

In Fig.1 we show the **$SU(5)$ -FUT** predictions for m_t and $m_b(M_Z)$ as a function of the unified gaugino mass M , for the two cases $\mu < 0$ and $\mu > 0$. We use the experimental value of the top quark pole mass as [100]²

$$m_t^{\text{exp}} = (173.2 \pm 0.9) \text{ GeV} . \quad (65)$$

The bottom mass is calculated at M_Z to avoid uncertainties that come from running down to the pole mass; the leading SUSY radiative corrections to the bottom and tau masses have been taken into account [103]. We use the following value for the bottom mass at M_Z [102],

$$m_b(M_Z) = (2.83 \pm 0.10) \text{ GeV} . \quad (66)$$

The bounds on the $m_b(M_Z)$ and the m_t mass clearly single out $\mu < 0$, as the solution most compatible with these experimental constraints.

Having found good agreement between the top and bottom quark predictions and the experimental data, we can apply further constraints on our model, **$SU(5)$ -FUT** with $\mu < 0$ (where the sign choice will be understood from now on). As additional constraints we consider the flavour observables $\text{BR}(b \rightarrow s\gamma)$ and $\text{BR}(B_s \rightarrow \mu^+\mu^-)$.

For the branching ratio $\text{BR}(b \rightarrow s\gamma)$, we take the value given by the Heavy Flavour Averaging Group (HFAG) is [104]

$$\text{BR}(b \rightarrow s\gamma) = (3.55 \pm 0.24_{-0.10}^{+0.09} \pm 0.03) \times 10^{-4} . \quad (67)$$

For the branching ratio $\text{BR}(B_s \rightarrow \mu^+\mu^-)$, the SM prediction is at the level of 3×10^{-9} , while we employ an upper limit of

$$\text{BR}(B_s \rightarrow \mu^+\mu^-) \lesssim 4.5 \times 10^{-9} \quad (68)$$

at the 95% C.L. [105]. This is in relatively good agreement with the recent direct measurement of this quantity by CMS and LHCb [106]. As we do not expect a sizable impact of the new measurement on our results, we stick for our analysis to the simple upper limit. Those limits will be applied for the further investigation of the impact of the LHC Higgs discovery on the predicted spectrum of the model.

The final observable we include into the discussion is the cold dark matter (CDM) density. It is well known that the lightest neutralino, being the lightest supersymmetric particle (LSP), is an excellent candidate for CDM [107,108]. Consequently one can demand that the lightest neutralino is indeed the LSP and parameters leading to a different LSP could be discarded.

² We did not include the latest LHC/Tevatron combination, leading to $m_t^{\text{exp}} = (173.34 \pm 0.76) \text{ GeV}$ [101], which would have a negligible impact on our analysis.

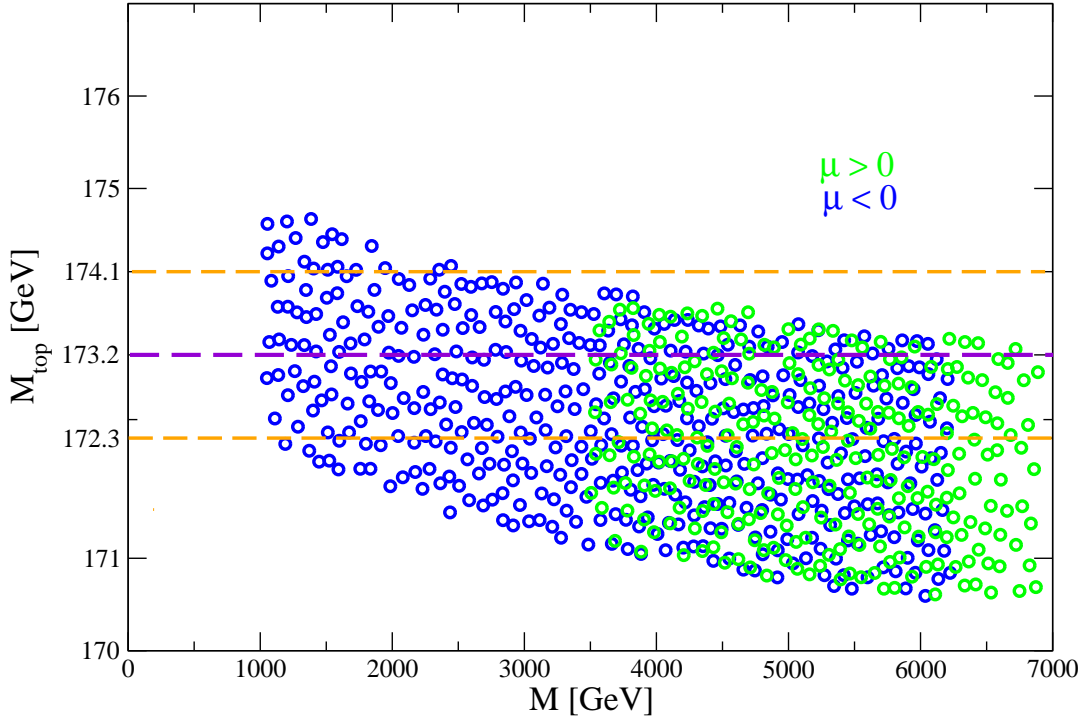
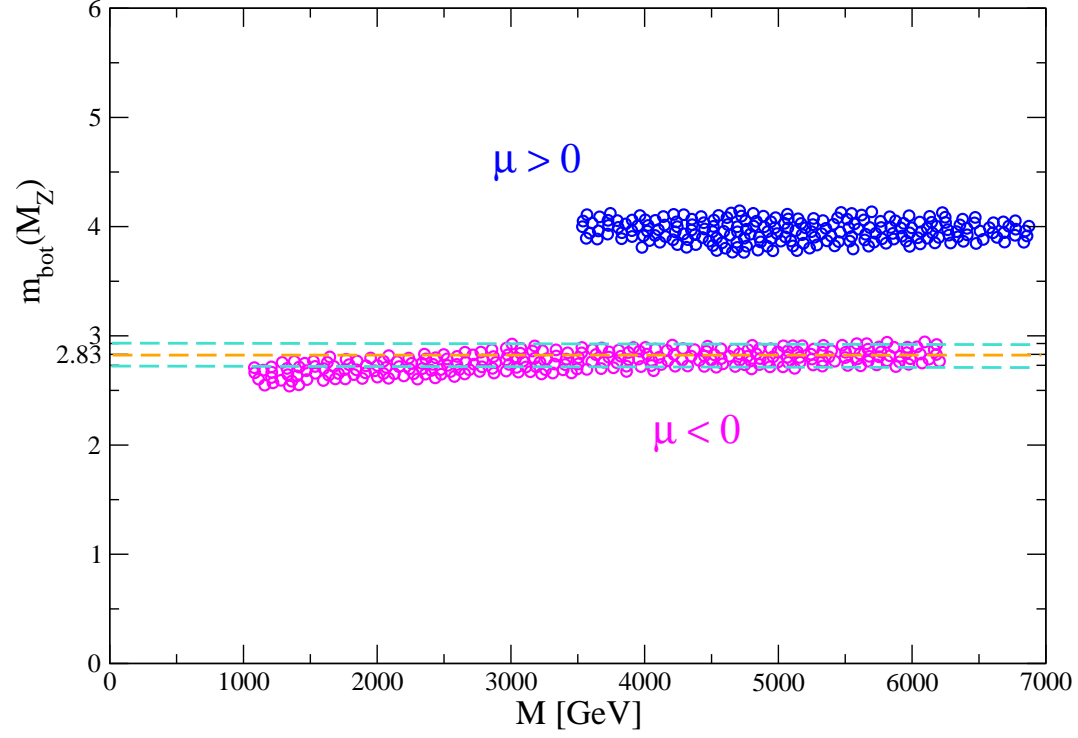


Figure 1: The bottom quark mass at the Z boson scale (upper) and top quark pole mass (lower plot) are shown as function of M for both models and both signs of μ .

The current bound, favored by a joint analysis of WMAP/Planck and other astrophysical and cosmological data, is at the 2σ level given by the range [109, 110],

$$\Omega_{\text{CDM}}h^2 = 0.1120 \pm 0.0112 . \quad (69)$$

We find that no model point of ***SU(5)-FUT*** (with $\mu < 0$) fulfills the strict bound of Eq. (69). (For our evaluation we have used the code **MicroMegas** [111, 112].) Consequently, on a more general basis a mechanism is needed in our model to reduce the CDM abundance in the early universe. This issue could, for instance, be related to another problem, that of neutrino masses. This type of masses cannot be generated naturally within the class of finite unified theories that we are considering in this paper, although a non-zero value for neutrino masses has clearly been established [102]. However, the class of FUTs discussed here can, in principle, be easily extended by introducing bilinear R-parity violating terms that preserve finiteness and introduce the desired neutrino masses [113–115]. R-parity violation [116–119] would have a small impact on the collider phenomenology presented here (apart from fact the SUSY search strategies could not rely on a ‘missing energy’ signature), but remove the CDM bound of Eq. (69) completely. The details of such a possibility in the present framework attempting to provide the models with realistic neutrino masses will be discussed elsewhere. Other mechanisms, not involving R-parity violation (and keeping the ‘missing energy’ signature), that could be invoked if the amount of CDM appears to be too large, concern the cosmology of the early universe. For instance, “thermal inflation” [120] or “late time entropy injection” [121] could bring the CDM density into agreement with the WMAP measurements. This kind of modifications of the physics scenario neither concerns the theory basis nor the collider phenomenology, but could have a strong impact on the CDM derived bounds. (Lower values than the ones permitted by Eq. (69) are naturally allowed if another particle than the lightest neutralino constitutes CDM.)

We will briefly comment on the anomalous magnetic moment of the muon, $(g - 2)_\mu$, at the end of Sect. 5.

4 The light Higgs boson mass in *SU(5)-FUT*

Due to the fact that the quartic couplings in the Higgs potential are given by the SM gauge couplings, the lightest Higgs boson mass is not a free parameter, but predicted in terms of the other model parameters. Higher-order corrections are crucial for a precise prediction of M_h [122–125].

In the Feynman diagrammatic approach that we are following here, the higher-order corrected \mathcal{CP} -even Higgs boson masses are derived by finding the poles of the (h, H) -propagator matrix. The inverse of this matrix is given by

$$(\Delta_{\text{Higgs}})^{-1} = -i \begin{pmatrix} p^2 - m_{H,\text{tree}}^2 + \hat{\Sigma}_{HH}(p^2) & \hat{\Sigma}_{hH}(p^2) \\ \hat{\Sigma}_{hH}(p^2) & p^2 - m_{h,\text{tree}}^2 + \hat{\Sigma}_{hh}(p^2) \end{pmatrix} . \quad (70)$$

Determining the poles of the matrix Δ_{Higgs} in Eq. (70) is equivalent to solving the equation

$$\left[p^2 - m_{h,\text{tree}}^2 + \hat{\Sigma}_{hh}(p^2) \right] \left[p^2 - m_{H,\text{tree}}^2 + \hat{\Sigma}_{HH}(p^2) \right] - \left[\hat{\Sigma}_{hH}(p^2) \right]^2 = 0 . \quad (71)$$

The spectacular discovery of a Higgs boson at ATLAS and CMS, as announced in July 2012 [65, 66] can be interpreted as the discovery of the light \mathcal{CP} -even Higgs boson of the MSSM Higgs spectrum [126–137]. The current experimental average for the (SM) Higgs boson mass is

$$M_H^{\text{exp}} = 125.6 \pm 0.3 \text{ GeV} . \quad (72)$$

Adding a 3 (2) GeV theory uncertainty [122] for the Higgs boson mass calculation in the (MFV) MSSM we arrive at

$$M_h = 125.6 \pm 3.1 \text{ (2.1) GeV} \quad (73)$$

as our allowed range.

For the lightest Higgs mass prediction we used the code **FeynHiggs** [122, 138–142]. The evaluation of Higgs boson masses within **FeynHiggs** is based on the Feynman-diagrammatic calculation as discussed above. **FeynHiggs** has recently been updated to version 2.10.0, where the principal focus of the improvements has been to attain greater accuracy for large stop masses. The new version, **FeynHiggs** 2.10.0 [142] contains a resummation of the leading and next-to-leading logarithms of type $\log(m_{\tilde{t}}/m_t)$ in all orders of perturbation theory, which yields reliable results for $m_{\tilde{t}}, M_A \gg M_Z$. To this end the two-loop Renormalization-Group Equations (RGEs) [143, 144] have been solved, taking into account the one-loop threshold corrections to the quartic coupling at the SUSY scale: see [145] and references therein. In this way at n -loop order the terms

$$\sim \log^n(m_{\tilde{t}}/m_t), \quad \sim \log^{n-1}(m_{\tilde{t}}/m_t) \quad (74)$$

are taken into account. As we shall see, **FeynHiggs** 2.10.0 yields a larger estimate of M_h for stop masses in the multi-TeV range (as we find in our evaluations), and a correspondingly improved estimate of the theoretical uncertainty, as discussed in [142, 146] (and indicated in Eq. (73)).

The prediction for M_h of **$SU(5)$ -FUT** (with $\mu < 0$) is shown in Fig. 2 as a function of M in the range $1 \text{ TeV} \lesssim M \lesssim 8 \text{ TeV}$. All points fulfill the quark mass requirements, while the blue points in addition also fulfill the B -physics constraints. The lightest Higgs mass ranges in

$$M_h \sim 124 - 133 \text{ GeV} , \quad (75)$$

where larger masses could be reached for larger values of M . The main uncertainty for fixed M comes from the variation of the other soft scalar masses. As discussed above, to this value one has to add at least $\pm 2 \text{ GeV}$ coming from unknown higher order corrections [142, 146]. We have also included a small variation, due to threshold corrections at the GUT scale, of up to 5% of the FUT boundary conditions. Overall, M_h is found at somewhat higher values in comparison with our previous analyses [147–152]. This is clearly due to the newly included resummed logarithmic corrections.

The horizontal lines in Fig. 2 show the central value of the experimental measurement (solid), the $\pm 2.1 \text{ GeV}$ uncertainty (dashed) and the $\pm 3.1 \text{ GeV}$ uncertainty (dot-dashed). The requirement to obtain a light Higgs boson mass value in the correct range yields an upper limit on M of about 3 (4.5) TeV for $M_h = 125.6 \pm 2.1 \text{ (3.1) GeV}$. Naturally this also sets an upper limit on the low-energy SUSY masses as will be reviewed in the next section.

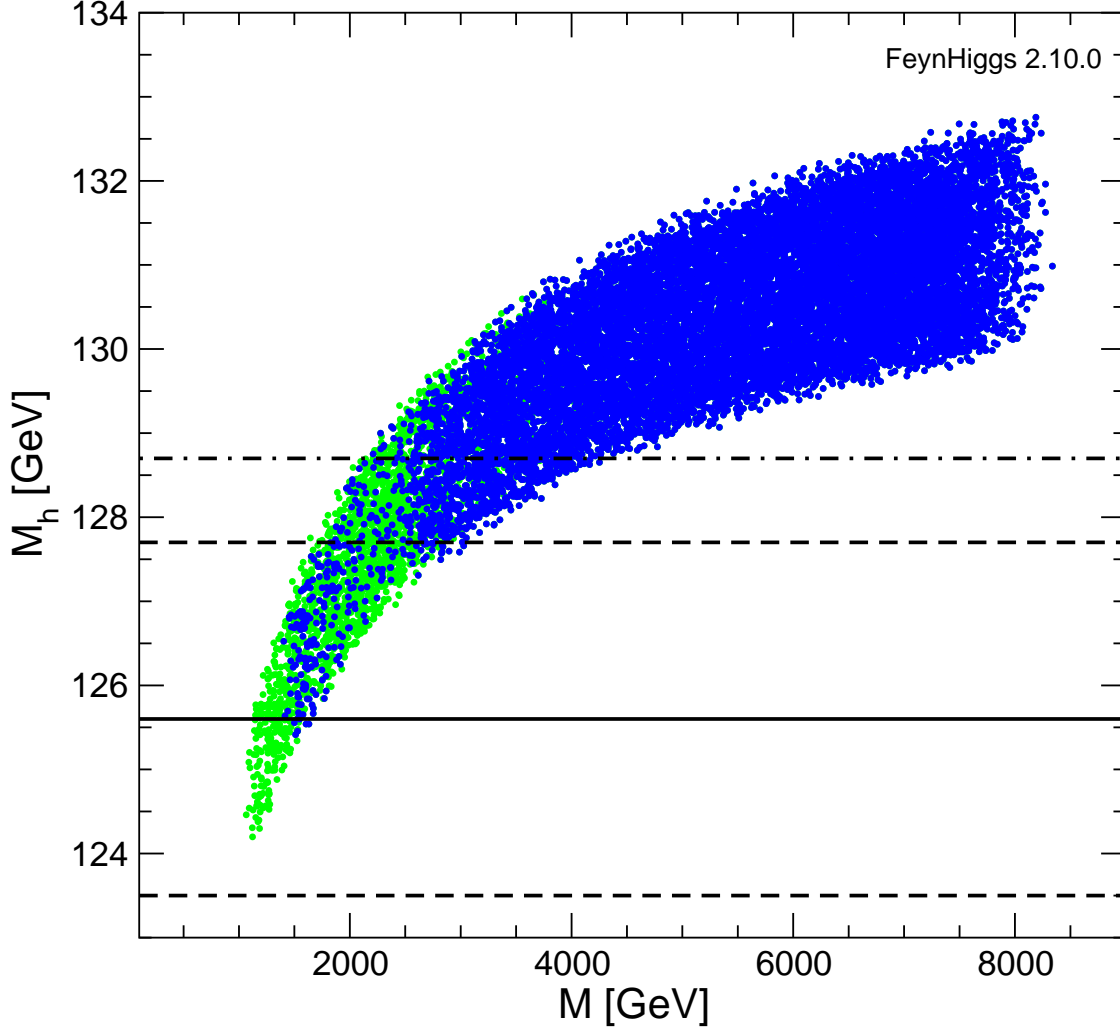


Figure 2: The lightest Higgs boson mass, M_h , as a function of M in the model $SU(5)$ -FUT (with $\mu < 0$). The blue points fulfill the B -physics constraints (see text).

5 $SU(5)$ -FUT spectrum predictions

In this section we analyze the predictions for the particle spectrum of $SU(5)$ -FUT (with $\mu < 0$), which is particular restricted by the requirement for the light Higgs boson mass, Eq. (73).

The full particle spectrum of model $SU(5)$ -FUT (with $\mu < 0$), compliant with quark mass constraints and the B -physics observables is shown in Fig. 3. In the upper (lower) plot we impose $M_h = 125.6 \pm 3.1$ (2.1) GeV. Including the Higgs mass constraints in general favors the lower part of the SUSY particle mass spectra. The “old” uncertainty estimate of ± 3.1 GeV permits SUSY masses in the multi-TeV range, which would remain unobservable at the LHC, the ILC or CLIC. Even the mass of the LSP (the lightest neutralino) could be above 2 TeV. On the other hand, using the “improved” theory estimate of ± 2.1 GeV³ results

³A more precise estimate requires a re-analysis of all sources of missing higher-order corrections and goes

in substantially lower upper limits of the SUSY mass spectrum. In this case the LSP ranges from about 0.6 TeV to about 1.5 TeV, so that it could be produced at CLIC (with $\sqrt{s} = 3$ TeV) via the process $e^+e^- \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 \gamma$. Also the second lightest neutralino as well as the two scalar taus could be in a mass range either accessible at the ILC (depending on the final center-of-mass energy) or at CLIC. The lightest scalar tau always turns out to be the Lightest Observable SUSY particle (LOSP). Similarly, the light chargino mass is found between ~ 1.2 TeV and ~ 2.6 TeV, with the second chargino mass slightly higher. The colored spectrum (scalar tops and bottoms, as well as the gluino) all have masses well above 1.7 TeV and are bounded from above by about 4 – 6 TeV. Only for the lighter part of the spectrum a discovery at the LHC might be possible. At the HL-LHC larger parts of the spectrum can be covered, but still part of the spectrum remains out of reach. The heavy Higgs boson masses range between ~ 1.2 TeV and ~ 5 TeV. The lower part could be covered at the LHC (in particular for the high $\tan\beta$ values found in our analysis) or later at CLIC, whereas the higher part could escape all current and planned collider experiments. The mass gap found for the masses of the heavy Higgs bosons stems from the fact that for intermediate values too *low* values of $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ are found, whereas in the very high-mass regime the SM value is recovered. Two numerical examples of mass spectra are shown in Tab. 1. The left part shows a representative (but still relatively light) spectrum, whereas the right part demonstrates a particularly light (but possible) parameter point. If such a light spectrum were realized, the colored particles are at the border of the discovery region at the LHC. Some uncolored particles like the scalar taus, the light chargino or the lighter neutralinos would be in the reach of a high-energy Linear Collider. The right part in Tab. 1, indicating a somewhat heavy spectrum, is mostly out of reach for the ILC, CLIC, or the HL-LHC. Depending on the reachable center-of-mass energy, the lightest electroweak particles might be borderline in the ILC reach. However, all in all, such kind of spectrum would effectively yield only a SM-like light Higgs boson visible at the various collider experiments.

Overall, the discovery of a Higgs boson, interpreted as the lightest MSSM Higgs boson, together with the refined M_h calculation allows to put substantially improved limits on the allowed particle spectrum. While in the older evaluations always large parts of the parameter space were out of reach for the LHC, the ILC and CLIC, the improved analysis nearly guarantees the discovery of one or more particles at the LHC or future e^+e^- colliders.

Finally, we note that with such a heavy SUSY spectrum the anomalous magnetic moment of the muon, $(g-2)_\mu$ (with $a_\mu \equiv (g-2)_\mu/2$), gives only a negligible correction to the SM prediction. The comparison of the experimental result and the SM value (based on the latest combination using e^+e^- data) [153]

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (28.7 \pm 8.0) \times 10^{-10} , \quad (76)$$

would disfavor ***SU(5)-FUT*** with $\mu < 0$ [154,155]. However, since the results would be very close to the SM results, the model has the same level of difficulty with the a_μ measurement as the SM.

far beyond the scope of this review.

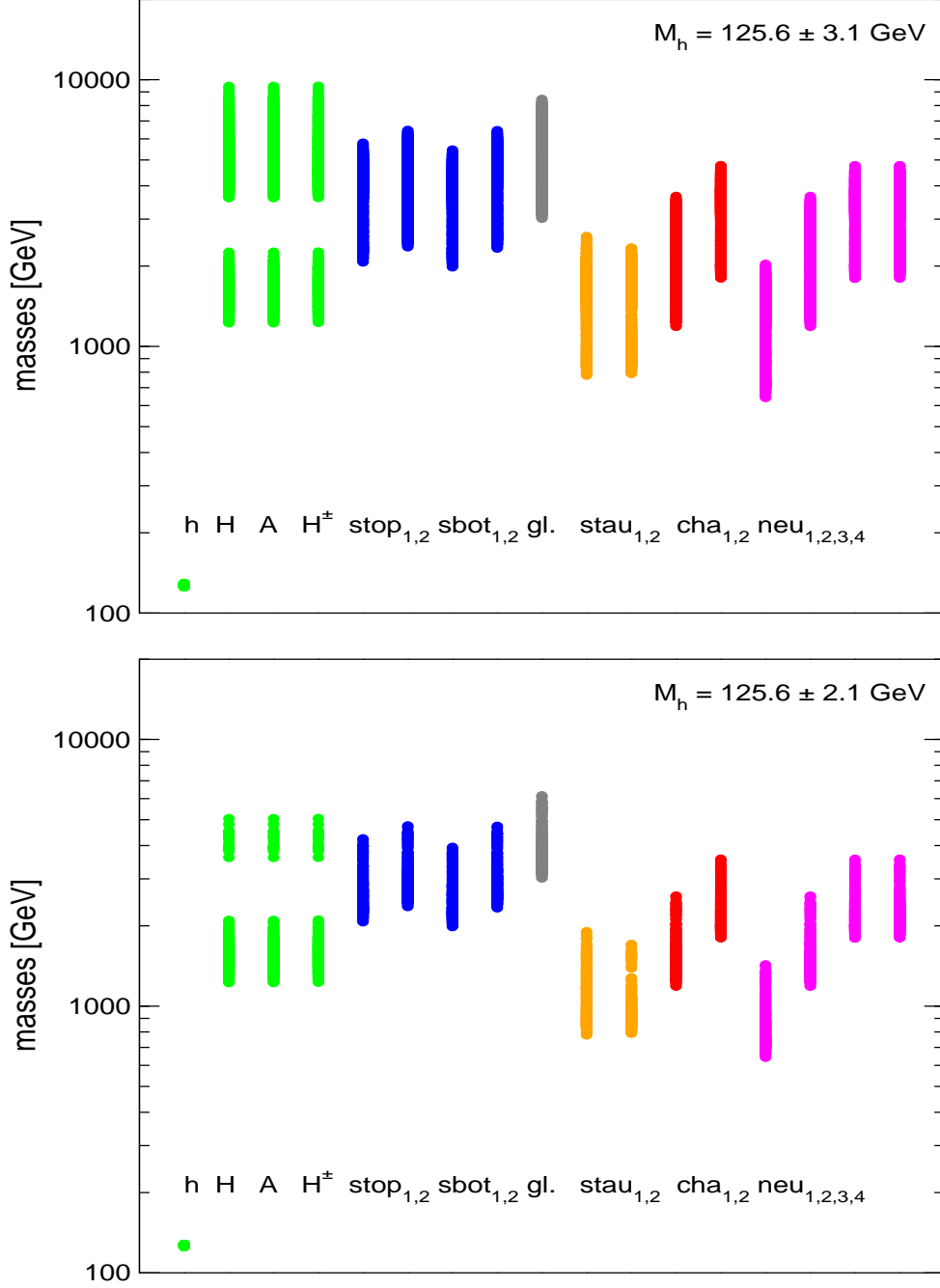


Figure 3: The upper (lower) plot shows the spectrum of $SU(5)$ -FUT (with $\mu < 0$) after imposing the constraint $M_h = 125.6 \pm 3.1$ (2.1) GeV. The points shown are in agreement with the quark mass constraints and the B -physics observables. The light (green) points on the left are the various Higgs boson masses. The dark (blue) points following are the two scalar top and bottom masses, followed by the lighter (gray) gluino mass. Next come the lighter (beige) scalar tau masses. The darker (red) points to the right are the two chargino masses followed by the lighter shaded (pink) points indicating the neutralino masses.

| | | | | | | | |
|--------------------------|-------|--------------------------|-------|--------------------------|-------|--------------------------|-------|
| $m_b(M_Z)$ | 2.74 | m_t | 172.1 | $m_b(M_Z)$ | 2.82 | m_t | 172.5 |
| M_h | 127.4 | M_A | 1514 | M_h | 128.6 | M_A | 4862 |
| M_H | 1514 | M_{H^\pm} | 1518 | M_H | 4861 | M_{H^\pm} | 4867 |
| $m_{\tilde{t}_1}$ | 2483 | $m_{\tilde{t}_2}$ | 2808 | $m_{\tilde{t}_1}$ | 4167 | $m_{\tilde{t}_2}$ | 4666 |
| $m_{\tilde{b}_1}$ | 2403 | $m_{\tilde{b}_2}$ | 2786 | $m_{\tilde{b}_1}$ | 3913 | $m_{\tilde{b}_2}$ | 4650 |
| $m_{\tilde{\tau}_1}$ | 892 | $m_{\tilde{\tau}_2}$ | 1089 | $m_{\tilde{\tau}_1}$ | 1593 | $m_{\tilde{\tau}_2}$ | 1756 |
| $m_{\tilde{\chi}_1^\pm}$ | 1453 | $m_{\tilde{\chi}_2^\pm}$ | 2127 | $m_{\tilde{\chi}_1^\pm}$ | 2547 | $m_{\tilde{\chi}_2^\pm}$ | 3515 |
| $m_{\tilde{\chi}_1^0}$ | 790 | $m_{\tilde{\chi}_2^0}$ | 1453 | $m_{\tilde{\chi}_1^0}$ | 1405 | $m_{\tilde{\chi}_2^0}$ | 2547 |
| $m_{\tilde{\chi}_3^0}$ | 2123 | $m_{\tilde{\chi}_4^0}$ | 2127 | $m_{\tilde{\chi}_3^0}$ | 3512 | $m_{\tilde{\chi}_4^0}$ | 3515 |
| $m_{\tilde{g}}$ | 3632 | | | $m_{\tilde{g}}$ | 6066 | | |

Table 1: A heavy (light) spectrum of a **$SU(5)$ -FUT** (with $\mu < 0$) parameter point is shown in the right (left) table. Both are compliant with the B physics constraints. All masses are in GeV.

6 Conclusions

A number of proposals and ideas have matured with time and have survived after careful theoretical studies and confrontation with experimental data. These include part of the original GUTs ideas, mainly the unification of gauge couplings and, separately, the unification of the Yukawa couplings, a version of fixed point behaviour of couplings, and certainly the necessity of SUSY as a way to take care of the technical part of the hierarchy problem. On the other hand, a very serious theoretical problem, namely the presence of divergencies in Quantum Field Theories (QFT), although challenged by the founders of QFT [156–158], was mostly forgotten in the course of developments of the field partly due to the spectacular successes of renormalizable field theories, in particular of the SM. However, as it was already mentioned in the Introduction, fundamental developments in theoretical particle physics are based in reconsiderations of the problem of divergencies and serious attempts to solve it. These include the motivation and construction of string and non-commutative theories, as well as $N = 4$ supersymmetric field theories [3, 4], $N = 8$ supergravity [5–9] and the AdS/CFT correspondence [2]. It is a thoroughly fascinating fact that many interesting ideas that have survived various theoretical and phenomenological tests, as well as the solution to the UV divergencies problem, find a common ground in the framework of $N = 1$ Finite Unified Theories, which we have described in the previous sections. From the theoretical side they solve the problem of UV divergencies in a minimal way. On the phenomenological side, since they are based on the principle of reduction of couplings (expressed via RGI relations among couplings and masses), they provide strict selection rules in choosing realistic models which lead to testable predictions. Finally, concerning the general program of reduction of couplings, we would like to quote the following citation: “... [this] looks at the moment as the only theoretically founded algorithm potentially able to decrease the number of parameters within the physically favored perturbative models.” [159].

We concentrated our examination on the predictions of one particular $SU(5)$ Finite Unified Theory, including the restrictions of third generation quark masses and B -physics observables. The model, **$SU(5)$ -FUT** (with $\mu < 0$), is consistent with all the phenomenological

constraints. Compared to our previous analysis [147], the new bound on $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ prefers a heavier (Higgs) spectrum and thus in general allows only a very heavy SUSY spectrum. The Higgs mass constraint, on the other hand, taking into account the improved M_h prediction for heavy scalar tops, favors the lower part of this spectrum, with SUSY masses ranging from ~ 600 GeV up to the multi-TeV level, where the lower part of the spectrum could be accessible at the ILC or CLIC. Taking into account the improved theory uncertainty evaluation some part of the electroweak spectrum should be accessible at future e^+e^- colliders. The colored spectrum, on the other hand, could easily escape the LHC searches; also at the HL-LHC non-negligible parts of the spectrum remain beyond the discovery reach.

The celebrated success of predicting the top-quark mass [23–28] has been extended to the predictions of the Higgs masses and the supersymmetric spectrum of the MSSM [147, 160]. Clear predictions for the discovery reach at current and future pp colliders as well as for future e^+e^- colliders result in somewhat more optimistic expectations compared to older analyses.

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